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Effect of Propeller Torque on Minimum-Control Airspeed

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Relations are developed for absolute static minimum-control airspeed, without the bank angle restrictions imposed on manned aircraft by regulatory agencies. Absolute static minimum-control airspeed can be important in the design of modern unmanned aerial vehicles, because autonomous computer controlled flight systems do not impose the same constraints as those imposed by human pilots. It is shown that, for a three-channel unmanned aerial vehicle, absolute static minimum-control airspeed always provides the critical constraint for static minimum-control airspeed. Even for the case of a conventional aircraft configuration with both rudder and ailerons, absolute static minimum-control airspeed can be a critical concern for small high-powered propeller-driven unmanned aerial vehicles.

Nomenclature

b_w = wingspan
$C_{\ell,\beta}$ = change in total aircraft rolling moment coefficient with sideslip angle
C_{ℓ,δ_a} = change in total aircraft rolling moment coefficient with aileron deflection angle
C_{ℓ,δ_r} = change in total aircraft rolling moment coefficient with rudder deflection angle
$C_{n,\beta}$ = change in total aircraft yawing moment coefficient with sideslip angle
C_{n,δ_a} = change in total aircraft yawing moment coefficient with aileron deflection angle
C_{n,δ_r} = change in total aircraft yawing moment coefficient with rudder deflection angle
$C_{Y,\beta}$ = change in total aircraft side force coefficient with sideslip angle
C_{Y,δ_a} = change in total aircraft side force coefficient with aileron deflection angle
C_{Y,δ_r} = change in total aircraft side force coefficient with rudder deflection angle
S_w = planform area of the main wing
V = airspeed
V_{mc} = minimum-control airspeed
V_{mcg} = ground-minimum-control airspeed
W = aircraft gross weight
β = sideslip angle, positive for sideslip to the right
$\Delta\ell_p$ = rolling moment increment contributed by the propulsion system at zero sideslip, positive to the right
Δn_p = yawing moment increment contributed by the propulsion system at zero sideslip, positive to the right
ΔY_p = side force increment contributed by the propulsion system at zero sideslip, positive to the right
δ_a = aileron deflection angle, left aileron positive
$\delta_{a_{\mathrm{sat}}}$ = effective aileron saturation angle
δ_r = rudder deflection angle, left rudder positive
$\delta_{r_{\mathrm{sat}}}$ = effective rudder saturation angle
ρ = air density
ϕ = bank angle, positive to the right
ϕ_{lim} = bank angle limit imposed by regulatory agency

Introduction

THE critical constraint on rudder size for multi-engine airplanes is typically based on the rudder deflection required to trim the airplane against asymmetric thrust with a critical engine inoperative and the remaining engine or engines at full power [1–6]. The rudder deflection required for such operational states increases as airspeed is decreased. This is because maximum thrust and the associated yawing moment typically decrease or remain constant with increasing airspeed, whereas the aerodynamic moments available from the controls are proportional to dynamic pressure. As airspeed is reduced and dynamic pressure decreases, the aerodynamic controls eventually lose the moment-generating capacity to hold against the asymmetric thrust. The airspeed at which either the rudder or aileron reaches its physical limit, called *saturation*, is referred to as the *static minimum-control airspeed*. Usually, the rudder provides the critical constraint on V_{mc}.

For aircraft powered by jet engines, which generate only slight inherent yawing and rolling moments, the critical engine used for evaluating V_{mc} is either of the two outboard engines. For propeller-driven aircraft, yawing and rolling moments are generated directly by the propellers and from the asymmetric slipstream flow over the fuselage and/or the wing and tail. For such aircraft, more careful thought is required to discern which of the two outboard engines is the critical one. The result of such analysis depends on the direction of propeller rotation.

Minimum-control airspeed is typically a concern because of the challenges associated with engine failure on takeoff or landing. If the rudder is inadequately sized, minimum-control airspeed will set the lower limit for both takeoff and landing speeds. Even if the wing with high-lift devices deployed is capable of flying at much lower airspeeds, pilots will avoid flying a multi-engine airplane at airspeeds below V_{mc}.

Operationally, the difficulties associated with maintaining control under conditions of engine failure at low airspeeds have killed pilots in airplanes that otherwise had excellent reputations, notably the P-38 Lightning and F-14 Tomcat. Consequently, V_{mc} is the fundamental reason that regulatory agencies require a separate pilot's license for multi-engine airplanes.

Static V_{mc} is only one of several minimum-control airspeeds. Federal Aviation Regulation (FAR) 23.149 [7] stipulates that V_{mc} may not exceed 1.2 times the stall speed at maximum takeoff weight.

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The definition used in the FAR describes a dynamic maneuver from maximum power in which a throttle chop to idle is performed on the critical engine. The pilot must then neutralize any resultant rates and return the airplane to a steady-heading sideslip with no more than 5 deg of bank, 150 lbf of rudder pedal force, and 20 deg of heading change. Flight-test literature [8] describes this as *dynamic* V_{mc} as opposed to the static V_{mc} previously described. Dynamic V_{mc} is more demanding than static V_{mc}, because the rudder must overcome not only the thrust asymmetry, but also any angular rates that may have developed as a result of the pilot's reaction time. Dynamic V_{mc} prediction requires high-fidelity man-in-the-loop simulation, because pilot interaction with both the displays and controls is an integral element in the airplane's resultant behavior.

The roll-control mechanization exerts surprising influence on dynamic V_{mc}. During an engine failure at high power, the resultant roll rate may actually mask the underlying yaw rate, causing the pilot to respond with aileron, rather than rudder. If the ailerons exhibit significant adverse yaw, countering the roll rate with aileron may actually erode control, because the adverse yaw will add to the yawing moment from asymmetric power. For example, dynamic V_{mc} for late-model F/A-18 Hornets was lowered during development by decreasing the aileron authority available to the pilot at certain low-speed conditions, mitigating severe adverse yaw [9].

Some airplanes also have a ground-minimum-control airspeed, usually denoted V_{mcg}. This is the minimum airspeed at which the rudder has adequate control power to maintain runway centerline within some tolerance, in the event of an engine failure during the takeoff ground roll. As with dynamic V_{mc}, prediction of V_{mcg} depends upon high-fidelity simulation, followed by flight-test validation.

Minimum-control airspeed testing is among the most hazardous of flight-test operations because the test team must probe the boundaries of controllability. Because of the strong sensitivity of thrust to density altitude, V_{mc} testing must be done at low altitude with little margin for surprise [9].

Estimation of Static V_{mc} for Conventional Piloted Airplanes

The requirements for lateral trim at constant airspeed, heading, and altitude are simply zero summations for the total side force and two lateral moments. The aerodynamic side force produced on the skin of the aircraft must balance the spanwise component of aircraft weight as well as any propulsive side force. Similarly, the aerodynamic rolling and yawing moments produced on the aircraft's skin must balance any lateral moments produced by the propulsion system. For small angles, the side force and both lateral moments are linear functions of the sideslip angle and lateral control surface deflections. Also for small bank angles, the spanwise component of aircraft weight can be approximated as a linear function of the bank angle expressed in radians, that is, $W \sin\phi \cong W\phi$ (see Fig. 1). Thus, the small-angle lateral trim requirements for flight at constant airspeed, heading, and altitude can be written in dimensionless form as

$$\begin{bmatrix} C_{Y,\beta} & C_{Y,\delta_a} & C_{Y,\delta_r} \\ C_{\ell,\beta} & C_{\ell,\delta_a} & C_{\ell,\delta_r} \\ C_{n,\beta} & C_{n,\delta_a} & C_{n,\delta_r} \end{bmatrix} \begin{Bmatrix} \beta \\ \delta_a \\ \delta_r \end{Bmatrix} + \begin{Bmatrix} (\Delta C_Y)_p \\ (\Delta C_\ell)_p \\ (\Delta C_n)_p \end{Bmatrix} + \begin{Bmatrix} C_W \\ 0 \\ 0 \end{Bmatrix} \phi = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

where the subscript p indicates a contribution from the aircraft's propulsion system at zero sideslip and

$$(\Delta C_Y)_p \equiv \frac{\Delta Y_p}{\frac{1}{2}\rho V^2 S_w}, \qquad (\Delta C_\ell)_p \equiv \frac{\Delta \ell_p}{\frac{1}{2}\rho V^2 S_w b_w}$$

$$(\Delta C_n)_p \equiv \frac{\Delta n_p}{\frac{1}{2}\rho V^2 S_w b_w}, \qquad C_W \equiv \frac{W}{\frac{1}{2}\rho V^2 S_w}$$

Examination of Eq. (1) reveals that, even with the weight, dynamic pressure, and all contributions from the propulsion system known, these lateral trim relations are underdetermined without some additional constraint on the four variables β, δ_a, δ_r, and ϕ. This means that static lateral trim can be attained with many different combinations of sideslip and bank angle. For typical multi-engine airplanes with one engine inoperative, the rudder deflection required to statically trim the airplane against asymmetric thrust can be substantially reduced by banking the airplane into the operating engine and allowing it to sideslip in the direction of that engine. However, there is a limit to how large a bank angle can be comfortably used for this purpose, particularly in close proximity to the ground. For the purpose of design and certification of conventional piloted airplanes, the magnitude of the bank angle is typically limited to no more than 5 deg [7,8].

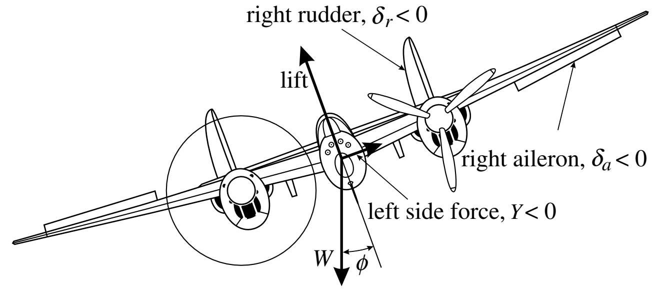

Fig. 1 Balancing the side force with one engine inoperative.

For analytically estimating the minimum-control airspeed, Eq. (1) can be conveniently rearranged as

$$\begin{Bmatrix} \beta/C_W \\ \delta_a/C_W \\ \delta_r/C_W \end{Bmatrix} = \begin{bmatrix} C_{Y,\beta} & C_{Y,\delta_a} & C_{Y,\delta_r} \\ C_{\ell,\beta} & C_{\ell,\delta_a} & C_{\ell,\delta_r} \\ C_{n,\beta} & C_{n,\delta_a} & C_{n,\delta_r} \end{bmatrix}^{-1} \begin{Bmatrix} -\Delta Y_p/W - \phi \\ -\Delta \ell_p/(b_w W) \\ -\Delta n_p/(b_w W) \end{Bmatrix} \tag{2}$$

With the lateral propulsive force and moments known and the bank angle fixed at its limiting value, this linear system can be used to evaluate the unknown angles β, δ_a, and δ_r divided by the weight coefficient C_W. For typical multi-engine airplanes with one engine inoperative, the sign for the bank angle used in Eq. (2) is that for a bank in the direction of the operating engine. Notice that, with the aerodynamic coefficients, gross weight, and propulsive contributions fixed, the trim angles per unit weight coefficient are independent of the unknown airspeed.

Because the minimum-control airspeed could be constrained by saturation of either the ailerons or rudder, both possibilities must be considered. Once the trim angles per unit weight coefficient have been determined from Eq. (2), the aileron- and rudder-limited minimum-control airspeeds for the specified bank angle can be, respectively, evaluated from the weight coefficient definition and the known control saturation angles

$$(V_{\mathrm{mc}})_{\mathrm{aileron}} = \sqrt{\frac{2W(\delta_a/C_W)}{\rho S_w \delta_{a_{\mathrm{sat}}}}} \tag{3}$$

$$(V_{\mathrm{mc}})_{\mathrm{rudder}} = \sqrt{\frac{2W(\delta_r/C_W)}{\rho S_w \delta_{r_{\mathrm{sat}}}}} \tag{4}$$

The saturation angles used in Eqs. (3) and (4) are those with the same sign as the corresponding trim angle per unit weight coefficient. In general, the actual minimum-control airspeed for the specified bank angle will be the larger of these two values. For typical piloted multi-engine airplanes, static V_{mc} is rudder limited.

The above formulation is based on linear relations between control moments and control deflections. While this is reasonable for control deflections having a magnitude less than about 10 deg, linearity commonly breaks down before saturation. Because control power typically decreases as saturation is approached, this formulation will underestimate the minimum-control airspeed if the actual maximum control surface deflection angles are used for saturation.

Improved results are obtained if effective control surface saturation angles are used. These effective angles are defined so that, when they are multiplied by the linear control power, the product is equal to the actual moment coefficient at saturation.

To examine how the aileron- and rudder-limited minimum-control airspeeds depend on bank angle for a propeller-driven multi-engine airplane, consider the P-38 Lightning shown in Fig. 1. For this airplane we shall use the following approximate values:

$$S_w = 327.5 \text{ ft}^2, \qquad b_w = 52 \text{ ft}, \qquad W = 15,480 \text{ lbf}$$
$$C_{Y,\beta} = -0.512, \qquad C_{\ell,\beta} = -0.025, \qquad C_{n,\beta} = 0.083$$
$$C_{Y,\delta_a} \cong 0.000, \qquad C_{\ell,\delta_a} = -0.131, \qquad C_{n,\delta_a} = 0.013$$
$$C_{Y,\delta_r} = 0.115, \qquad C_{\ell,\delta_r} = 0.006, \qquad C_{n,\delta_r} = -0.052$$
$$\delta_{a_{\text{sat}}} = \pm 20 \text{ deg}, \qquad \delta_{r_{\text{sat}}} = \pm 30 \text{ deg}$$

The P-38 is powered by two 1,425-horsepower Allison V-1710 engines turning 138-in., three-blade, counter-rotating Curtiss Electric propellers with 2:1 gear reduction. The propeller on the pilot's right has right-hand rotation and the propeller on the left has left-hand rotation. Each propeller axis is offset from the plane of symmetry by 8 ft. With the left-hand engine inoperative and full power applied to the right engine, the operating propeller is turning at 1,500 rpm and produces rolling and yawing moments of −4,990 and −22,500 ft · lbf, respectively. Using these values in Eqs. (2–4) produces the results shown in Fig. 2.

A number of important observations can be made from examination of Fig. 2. First we recognize that lateral trim at constant airspeed, heading, and altitude can be maintained only for combinations of airspeed and bank angle that lie within the shaded region above both the aileron- and rudder-limited minimum-control airspeeds. As a result of the large propulsive yawing moment, V_{mc} is rudder limited at zero-bank angle. Because the propulsive rolling and yawing moments are both negative for this operating condition, positive rolling and yawing moments must be supplied by the controls (i.e., right aileron $\delta_a < 0$, and right rudder $\delta_r < 0$). For this engine-out operating condition, the right rudder required to statically trim the airplane against the asymmetric thrust from the right engine can be reduced from that needed for the zero-bank solution by increasing the bank angle to the right and allowing the airplane to sideslip to the right. When the airplane is statically trimmed with some sideslip to the right, a yawing moment to the right is generated through the yaw stiffness $C_{n,\beta}$. This yawing moment helps balance the propulsive yawing moment to the left, thereby reducing the right rudder needed to statically trim the airplane. For this example, static V_{mc} is 255 ft/s with no bank angle, but using a bank angle of 5 deg reduces the static V_{mc} to 184 ft/s with a sideslip angle to the right of 4.74 deg. We also see that increasing the bank angle somewhat beyond 5 deg would reduce V_{mc} even further. As the bank angle to the right continues to increase, the sideslip angle to the right also increases and static V_{mc} is reduced, until either the ailerons become saturated or the vertical stabilizers stall. The minimum-control airspeed at which both the rudder and ailerons are saturated is denoted here as the *absolute minimum-control airspeed*. Because Eqs. (2–4) are based on a linear relation between the sideslip angle and the lateral force and moments, the results shown in Fig. 2 do not account for vertical stabilizer stall.

The direction of propeller rotation can significantly influence static minimum-control airspeed. For instance, if the direction of propeller rotation used for the previous example is reversed to produce a positive propulsive rolling moment of 4,990 ft · lbf while keeping the propulsive yawing moment and all other parameters fixed, the results shown in Fig. 3 are obtained. Notice that this change in the direction of propeller rotation reduces static V_{mc} with a 5 deg bank from 184 ft/s to 176 ft/s and reduces the absolute static V_{mc} to well below stall. It is interesting to note that the XP-38 prototype was actually built with this propeller rotation.

The difference between the results shown in Figs. 2 and 3 was produced by simply changing the sign of the propulsive rolling moment with no change in the propulsive yawing moment. In reality, changing the direction of propeller rotation can also affect the propulsive yawing moment. At a positive angle of attack, a right-hand turning propeller produces an inherent yawing moment to the left and a left-hand turning propeller produces an inherent yawing moment to the right. Thus, because the propeller angle of attack is typically positive at low airspeeds, the magnitude of the net propulsive yawing moment is reduced when the working propeller on the pilot's right has left-hand rotation. When this effect is added to the results shown in Fig. 3, the net propulsive yawing moment to the left is reduced by about 1,100 ft · lbf and static V_{mc} with a 5 deg bank is further reduced to about 167 ft/s.

Although data used in the previous examples are approximate, the results are reasonable. According to the P-38 pilot's flight operating instructions [10], on failure of one engine, the pilot must "hold 125 mph or more (at least 160 mph preferred)." If one engine fails below 120 mph the pilot is to "close both throttles and land straight ahead, retracting the landing gear if it is not possible to land on the runway." Historical references indicate that some of the best pilots would not apply full power to the P-38 in single-engine operation at speeds below 150 mph. In 1942, many pilots transitioned to the Lightning with no previous twin-engine experience. Occasionally, pilots were killed when one engine failed on takeoff and the airplane rolled over on its back and into the ground. While otherwise the P-38

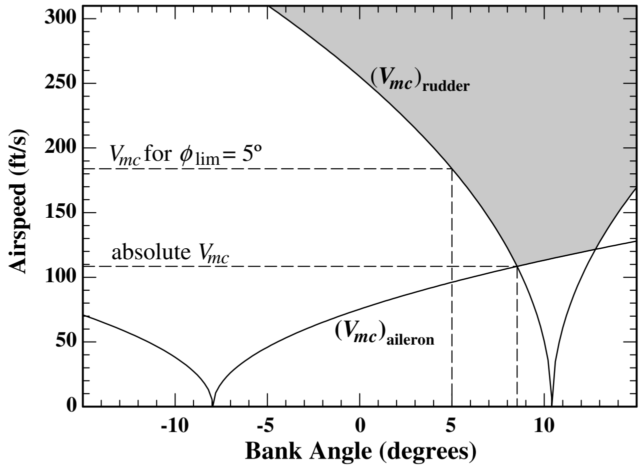

Fig. 2 The aileron- and rudder-limited minimum-control airspeeds as a function of bank angle for the P-38 Lightning with the engine on the pilot's right turning the right-hand propeller at full power and the engine on the pilot's left inoperative.

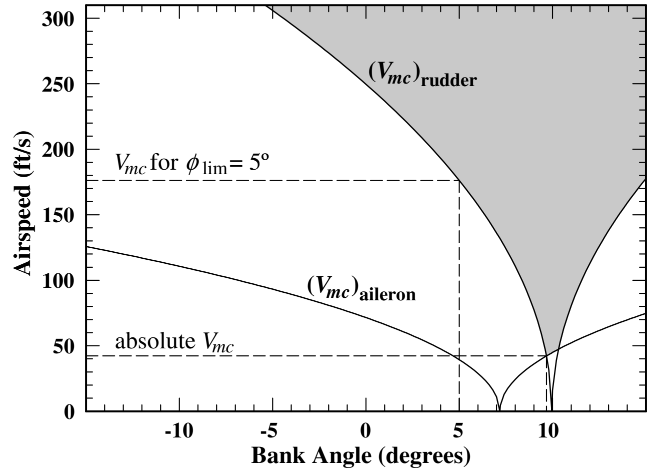

Fig. 3 The aileron- and rudder-limited minimum-control airspeeds as a function of bank angle for the XP-38 with the engine on the pilot's right turning the left-hand propeller at full power and the engine on the pilot's left inoperative.

had an excellent reputation, some of the top Lightning pilots used a takeoff technique that reflected their distrust in the Allison engines. They would keep the airplane on the ground to 150 mph, even though the manual recommended liftoff at 90 to 100 mph. Usually, no attempt was made to keep the P-38 above V_{mc} on landing with one engine out, because a touchdown speed above 100 mph simply required too much runway. Using the approach procedure recommended for single-engine operation, once below 500 feet the P-38 had to be landed, because the airspeed was too low to make a go-around on one engine. This was not considered to be a design flaw in the Lightning, because the engine-out options available to pilots of single-engine fighters were even more limited.

Estimation of Absolute Static V_{mc}

For conventional piloted aircraft, V_{mc} is governed primarily by the rudder deflection required to trim an airplane against the propulsive yawing moment resulting from engine failure in multi-engine airplanes. Still, we should note from Eqs. (1) and (2), as well as from comparison of Figs. 2 and 3, that a propulsive rolling moment also contributes to V_{mc}. For propeller-driven aircraft with high thrust-to-weight ratio, this can be significant. If propeller torque is large enough, V_{mc} can be a consideration even in single-engine aircraft. This can become more important in the design of small unmanned aerial vehicles (UAV). Limits on the thrust that can be developed by a propeller are proportional to the propeller disk area and propeller radius. Thus, if s is the length scale for an airplane, these thrust limits are proportional to s^2 and s. Because weight is proportional to s^3, thrust-to-weight ratio limits for propeller-driven aircraft are proportional to s^{-1} and s^{-2}. As airplanes are scaled down in size, very large propeller thrust-to-weight ratios become possible and torque related V_{mc} issues can become critical.

For a propeller-driven airplane without balanced counter-rotating propellers, a rolling moment must be generated by the controls to trim the airplane against the rolling moment resulting from propeller torque. If propeller torque is large enough, then roll control rather than yaw control becomes the critical constraint and V_{mc} becomes aileron limited at zero bank angle. While Eqs. (2–4) are useful for estimating static V_{mc} for the typical case where a yawing moment is the primary lateral contribution of the propulsion system, in cases where the primary lateral propulsive contribution is a rolling moment, a different method of analysis is needed.

The rudder deflection required for V_{mc} can typically be reduced by increasing the bank angle in the direction of the required rudder deflection (i.e., increasing the bank angle to the right when right rudder is required and increasing the bank angle to the left when left rudder is required). If the primary lateral propulsive contribution is a yawing moment, static V_{mc} is typically rudder limited at zero bank angle and this minimum-control airspeed can be reduced by increasing the bank in the direction of the rudder deflection. In such cases, the useful static minimum-control airspeed typically occurs at the maximum allowable bank angle as predicted by Eqs. (2–4) and the absolute minimum-control airspeed shown in Figs. 2 and 3 typically occurs at a bank angle that is greater than the maximum allowable bank angle. On the other hand, when a rolling moment is the primary lateral contribution of the propulsion system, static V_{mc} is usually aileron limited at zero bank angle and the absolute minimum-control airspeed commonly occurs at a bank angle that is less than the maximum allowable bank angle.

As can be seen in Figs. 2 and 3, the absolute minimum-control airspeed with no bank angle restriction is that for which aileron and rudder are both saturated simultaneously. For the purpose of evaluating this absolute V_{mc}, Eq. (1) can be rearranged as

$$\begin{bmatrix} C_{Y,\beta} & \Delta Y_p + W\phi \\ C_{\ell,\beta} & \Delta\ell_p/b_w \\ C_{n,\beta} & \Delta n_p/b_w \end{bmatrix} \begin{Bmatrix} \beta \\ 1/(\frac{1}{2}\rho V_{mc}^2 S_w) \end{Bmatrix} = -\begin{Bmatrix} C_{Y,\delta_a}\delta_a + C_{Y,\delta_r}\delta_r \\ C_{\ell,\delta_a}\delta_a + C_{\ell,\delta_r}\delta_r \\ C_{n,\delta_a}\delta_a + C_{n,\delta_r}\delta_r \end{Bmatrix} \tag{5}$$

With the propulsive lateral force and moments known and the control deflections fixed at the saturation values, the second and third of these relations can be solved algebraically for the airspeed and sideslip angle. These results can then be used in the first relation to obtain the bank angle. This yields

$$V_{\mathrm{mc}} = \sqrt{\frac{2(C_{\ell,\beta}\Delta n_p - C_{n,\beta}\Delta\ell_p)/(\rho S_w b_w)}{(C_{n,\beta}C_{\ell,\delta_a} - C_{\ell,\beta}C_{n,\delta_a})\delta_{a_{\mathrm{sat}}} + (C_{n,\beta}C_{\ell,\delta_r} - C_{\ell,\beta}C_{n,\delta_r})\delta_{r_{\mathrm{sat}}}}} \tag{6}$$

$$\beta = -\frac{(\Delta C_n)_p + C_{n,\delta_a}\delta_{a_{\mathrm{sat}}} + C_{n,\delta_r}\delta_{r_{\mathrm{sat}}}}{C_{n,\beta}} = -\frac{(\Delta C_\ell)_p + C_{\ell,\delta_a}\delta_{a_{\mathrm{sat}}} + C_{\ell,\delta_r}\delta_{r_{\mathrm{sat}}}}{C_{\ell,\beta}} \tag{7}$$

$$\phi = -\frac{(\Delta C_Y)_p + C_{Y,\beta}\beta + C_{Y,\delta_a}\delta_{a_{\mathrm{sat}}} + C_{Y,\delta_r}\delta_{r_{\mathrm{sat}}}}{C_W} \tag{8}$$

If the bank angle predicted from Eq. (8) is less than the limiting bank angle, static V_{mc} can be estimated from Eq. (6). This will commonly be the case when the V_{mc} predicted from Eqs. (2–4) is aileron limited at zero bank angle.

For example, consider a small UAV designed and built at Utah State University. For this airplane we will use the following approximate values:

$$S_w = 5.88\ \mathrm{ft}^2, \qquad b_w = 7.43\ \mathrm{ft}, \qquad W = 9.0\ \mathrm{lbf}$$
$$C_{Y,\beta} = -0.720, \qquad C_{\ell,\beta} = -0.065, \qquad C_{n,\beta} = 0.177$$
$$C_{Y,\delta_a} = -0.034, \qquad C_{\ell,\delta_a} = -0.075, \qquad C_{n,\delta_a} = 0.006$$
$$C_{Y,\delta_r} = 0.231, \qquad C_{\ell,\delta_r} = 0.014, \qquad C_{n,\delta_r} = -0.074$$
$$\delta_{a_{\mathrm{sat}}} = \pm 15\ \mathrm{deg}, \qquad \delta_{r_{\mathrm{sat}}} = \pm 25\ \mathrm{deg}$$

The airplane is powered by a single electric motor that develops 0.95 hp turning a propeller at 3,100 rpm. Neglecting slipstream effects and any yawing moment produced by the centered propeller, its only lateral contribution is a rolling moment $\Delta\ell_p = -1.61\ \mathrm{ft\cdot lbf}$. At standard sea level, using this value in Eqs. (6–8) yields $V_{\mathrm{mc}} = 35.4\ \mathrm{ft/s}$, $\beta = -9.94$ deg, and $\phi = -1.84$ deg. This minimum-control airspeed is above the stall speed for this aircraft of 31 ft/s. To reduce static V_{mc} below stall for this aircraft, the aileron control power would need to be increased by more than 50%.

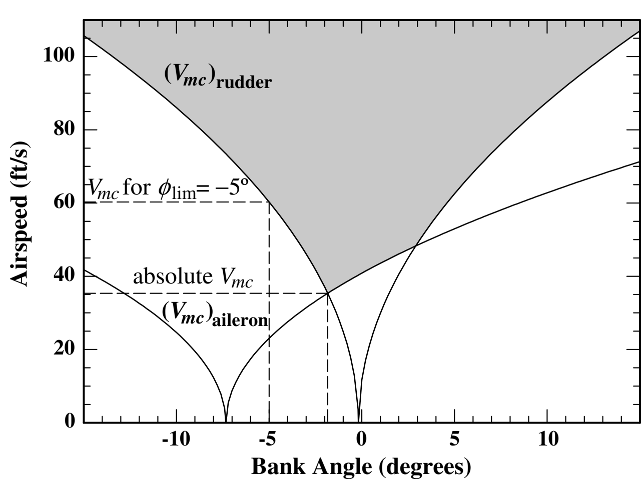

Fig. 4 The aileron- and rudder-limited minimum-control airspeeds as a function of bank angle for a small single-engine UAV with a right-hand turning propeller producing a propulsive rolling moment to the left and no propulsive yawing moment.

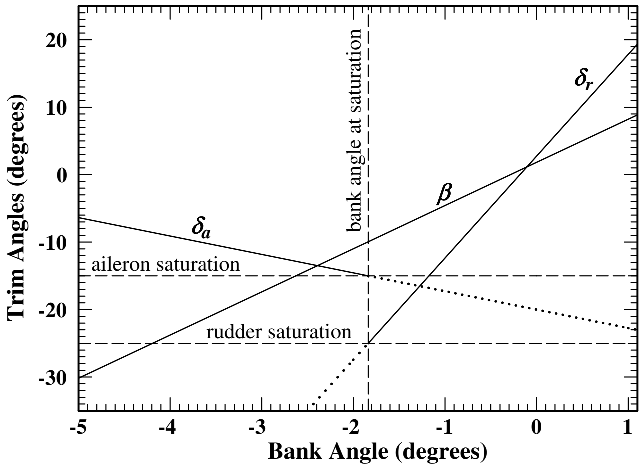

Fig. 5 Sideslip and control angles required for lateral trim at V_{mc} as a function of bank angle for a small single-engine UAV with a right-hand turning propeller producing a propulsive rolling moment to the left and no propulsive yawing moment.

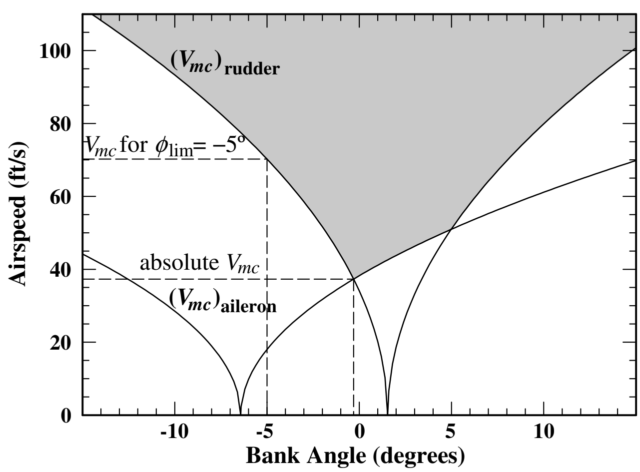

Fig. 6 The aileron- and rudder-limited minimum-control airspeeds as a function of bank angle for a small single-engine UAV with a right-hand turning propeller producing a propulsive rolling moment to the left and a propulsive yawing moment to the left.

For this aircraft, the aileron- and rudder-limited minimum-control airspeeds predicted from Eqs. (2–4) are plotted as a function of bank angle in Fig. 4. Notice that the rudder-limited minimum-control airspeeds for bank angles of ± 5 deg are significantly above the absolute V_{mc}, which is predicted from Eqs. (6–8) to occur at a bank angle of -1.84 deg. At this absolute minimum-control airspeed of 35.4 ft/s, the sideslip and control angles required for lateral trim at constant airspeed, heading, and altitude are plotted as a function of bank angle in Fig. 5. Notice that, at this airspeed, increasing the bank angle to the left ($\phi < -1.84$ deg) would require more rudder deflection than is available at saturation. On the other hand, decreasing the bank angle to the left ($\phi > -1.84$ deg) requires more aileron deflection than is available at saturation. Because both rudder and aileron are saturated to maintain lateral trim at this airspeed, the bank angle to the left can neither be increased nor decreased, without increasing the airspeed. This is the definition of absolute minimum-control airspeed as presented here. Additional airspeed is needed to provide maneuverability.

When the axis of propeller rotation is at some angle of attack with the freestream, a rotating propeller produces a yawing moment as well as a rolling moment. At a positive angle of attack, a right-hand turning propeller produces a negative rolling moment and a negative yawing moment. A left-hand propeller produces positive rolling and yawing moments at a positive angle of attack. This added yawing moment will increase the minimum-control airspeed for a single-engine propeller-driven UAV. For example, Fig. 6 shows the results of adding a propulsive yawing moment of $\Delta n_p = -0.50$ ft · lbf to the computations used to obtain the results plotted in Fig. 4.

A special case, where Eq. (6) is always useful, is in the design of what is typically called a *three-channel* UAV. Such aircraft have only three controls. Most commonly these are throttle, elevator, and rudder (no ailerons). The only roll control is provided indirectly by the rudder's interaction with wing dihedral. The absence of control surfaces in the wings makes this three-channel design particularly attractive for a small UAV that must be stored compactly and assembled quickly. For such aircraft, V_{mc} can be a critical constraint even for single-engine designs, due to the contribution of propeller torque. Static V_{mc} is always aileron limited in such three-channel airplanes, because aileron saturation occurs at $\delta_{a_{\text{sat}}} = 0$. In this case, Eqs. (6–8) must be used to estimate static V_{mc}, because Eqs. (2–4) are in an indeterminate form for an aircraft without aileron control. This results from the fact that, with no aileron control and the propulsive lateral contributions fixed at any given airspeed, only one combination of rudder deflection, sideslip, and bank angle can be used to maintain lateral trim at constant airspeed, heading, and altitude.

Another approach to the three-channel UAV design is where only throttle, aileron, and elevator controls are employed (for instance AeroVironment's *DragonEye* and BAI's *Evolution*) [11]. Obviously in these cases, static V_{mc} is rudder limited, because rudder saturation occurs at $\delta_{r_{\text{sat}}} = 0$. Here again, Eqs. (2–4) are indeterminate and Eqs. (6–8) must be used to estimate static V_{mc}.

Conclusion

Aircraft developers, regulators, and operators have long recognized the vital importance of static and dynamic V_{mc} as a critical constraint associated with the asymmetric thrust resulting from engine failure in multi-engine airplanes. However, absolute static V_{mc}, as defined and developed in this paper, has not typically been a critical concern in the design or operation of conventional piloted aircraft. Concerns over the limitations of human pilots have placed it outside the bounds of practical aircraft design space. On the other hand, absolute static V_{mc} can be important in the design of modern UAVs, because autonomous computer controlled flight systems do not impose the same constraints as those imposed by human pilots. For three-channel UAVs, absolute static V_{mc} as predicted from Eqs. (6–8) always provides the critical constraint for static minimum-control airspeed, because Eqs. (2–4) are indeterminate for such aircraft designs. Even for the case of a conventional aircraft configuration with rudder and substantial ailerons, absolute static V_{mc} as predicted from Eqs. (6–8) can be a critical concern for small high-powered propeller-driven UAVs, including single-engine designs with centerline thrust.

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